1) DATA LA CURVA (γ, I) CON $I=[0,2\pi]$ E PARAMETRIZZAZIONE

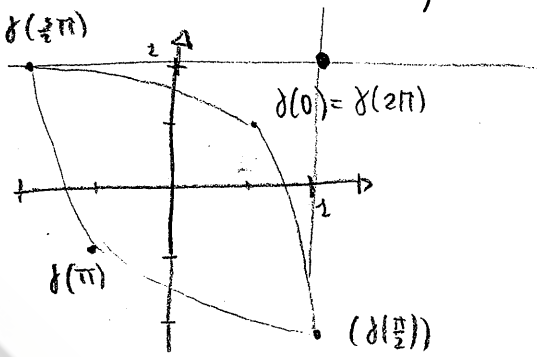
$$\gamma(t)=\left(\frac{1}{2}\cos t+\sin t,\ \frac{1}{2}\cos t-\sin t\right) \qquad t\in[0,2\pi]$$

(1)

CALCOLARE IL LAVORO LUNGO γ DAL CAMPO DI VETTORI

$$F(x,y)=\begin{pmatrix}\dfrac{x-y}{(x-1)^2+(y-1)^2}\\[2ex] \dfrac{x+y-2}{(x-1)^2+(y-1)^2}\end{pmatrix}$$

$\text{dom } F=\mathbb{R}^2\setminus\{x=1, y=1\}$



$$\gamma(0)=\left(\frac{1}{2},\frac{1}{2}\right)$$

$$\gamma(2\pi)=\left(\frac{1}{2},\frac{1}{2}\right)$$

$$\gamma\left(\frac{\pi}{2}\right)=(1,-1)$$

$$\gamma(\pi)=\left(-\frac{1}{2},-\frac{1}{2}\right)$$

$$\gamma\left(\frac{3}{2}\pi\right)=(-1,+1)$$

} CURVA CHIUSA

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$$\text{rot}(F)=\frac{\left((x-1)^2+(y-1)^2\right)-(x+y-2)\cdot 2(x-1)}{\left((x-1)^2+(y-1)^2\right)^2}-\frac{-\left((x-1)^2+(y-1)^2\right)-(x-y)\cdot 2(y-1)}{\left((x-1)^2+(y-1)^2\right)^2}=0 \quad \#$$

F È IRROTAZIONALE.

$$F(\gamma(t))=\begin{pmatrix}\dfrac{2\sin t}{\frac{\cos^2 t}{2}+2\sin^2 t-2\cos t+2}\\[2ex] \dfrac{\cos t-2}{\frac{\cos^2 t}{2}+2\sin^2 t-2\cos t+2}\end{pmatrix}$$

$$\gamma'(t)=\begin{pmatrix}-\frac{1}{2}\sin t+\cos t\\ -\frac{1}{2}\sin t-\cos t\end{pmatrix}$$

$$\int_0^{2\pi}\frac{-\sin^2 t+2\cos t\sin t-\frac{1}{2}\cos t\sin t-\cos^2 t+\sin t+2\cos t}{\frac{\cos^2 t}{2}+2\sin^2 t-2\cos t+2}\,dt$$

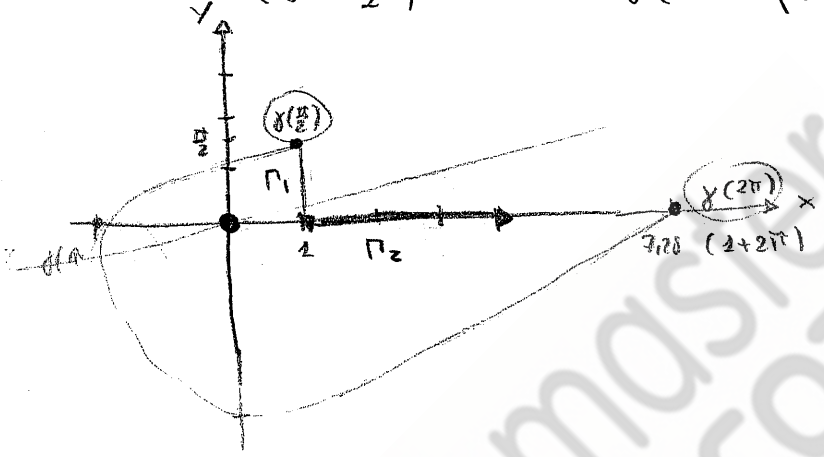
$$=\int\frac{\sin t+2\cos t-1+\frac{3}{2}\cos t\sin t}{\frac{\cos^2 t}{2}+2\sin^2 t-2\cos t+2}\,dt$$

LA CURVA $\gamma(t)$ È COMPLETAMENTE CONTENUTA NEL DOMINIO X, PERCIÒ POSSIAMO APPLICARE IL TEOREMA DEL ROTORE. (2)

$$L(F,\gamma) = \iint_U \operatorname{rot}(F)(x,y)\,dx\,dy = 0$$

$\gamma(t) = (1 + t\cos t,\ t\sin t) \qquad t \in [\frac{\pi}{2}, 2\pi]$

$\gamma(\frac{\pi}{2}) = (1, \frac{\pi}{2})$ (IN) $\qquad \gamma(2\pi) = (1+2\pi, 0)$ (FINE)



$\gamma(\pi) = (1-\pi, 0)$

$\gamma(\frac{3}{2}\pi) = (0, -\frac{3}{2}\pi)$

CALCOLA IL LAVORO DEL CAMPO DI VETTORI

$$F(x,y) = \begin{pmatrix} \frac{x-y}{x^2+y^2} \\ \frac{x+y}{x^2+y^2} \end{pmatrix} \qquad X = \operatorname{dom} F = \mathbb{R}^2 \setminus \{x^2+y^2=0\} \quad (0,0)$$

X NON È SEMPLICEMENTE CONNESSO

$$\operatorname{rot}(F) = \frac{(x^2+y^2) - (x+y)(2x)}{(x^2+y^2)^2} - \frac{-(x^2+y^2) - (x-y)(2y)}{(x^2+y^2)^2} = 0$$

F È IRROTAZIONALE.

X NON È SEMPLICEMENTE CONNESSO

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$$L(F,\gamma) = L(F,\gamma_1) + L(F,\gamma_2)$$

$\Gamma_1 = \{x = 1,\ 0 \le y \le \frac{\pi}{2}\} \qquad \gamma_1(t) = (1, t) \qquad t \in [0, \frac{\pi}{2}]$

$\gamma_1'(t) = (0,1) \qquad F(\gamma(t)) = \begin{pmatrix} \frac{1-t}{1+t^2} \\ \frac{1+t}{1+t^2} \end{pmatrix}$

$$L(F,\gamma_1) = \int_0^{\frac{\pi}{2}} \frac{1+t}{1+t^2}\,dt = \int \frac{1}{1+t^2} + \frac{1}{2}\int \frac{2t}{1+t^2}$$

$$\left| \arctan t + \frac{1}{2}\log(1+t^2) \right|_0^{\frac{\pi}{2}} = \arctan\frac{\pi}{2} + \frac{\log(1+\frac{\pi}{4}^2)}{2} \qquad (-)$$

$\Gamma_2 = \{ 1 \le x \le 1+2\pi \ , \ y = 0 \}$ $\gamma(t) = (t, 0)$ $t \in [1, 1+2\pi]$

$\gamma'(t) = (1, 0)$ $F(\gamma(t)) = \begin{pmatrix} \frac{1}{t} \\ \frac{1}{t} \end{pmatrix}$ ③

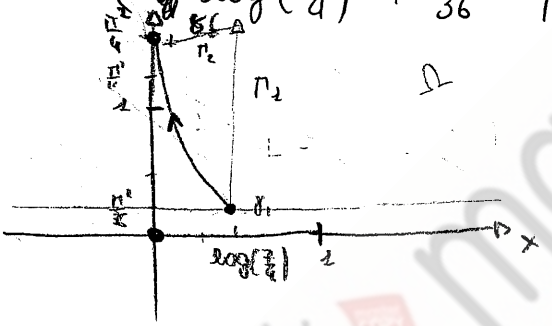
$$L(F, \gamma_2) = \int_1^{1+2\pi} \frac{1}{t}\, dt = \log(1+2\pi)$$

NO

$$L(F, \gamma) = \log(1+2\pi) - \arctan\frac{\pi}{2} - \frac{\log\left(1+\frac{\pi^2}{4}\right)}{2}$$

$\gamma(t) = (\log(1+\cos^2 t), t^2)$ $t \in [\frac{\pi}{6}, \frac{\pi}{2}]$

$\gamma(\frac{\pi}{6}) = (\log(\frac{7}{4}), \frac{\pi^2}{36})$ $\gamma(\frac{\pi}{2}) = (0, \frac{\pi^2}{4})$ $\gamma(\frac{\pi}{4}) = (\log\frac{3}{2}, \frac{\pi^2}{16})$



$$F(x,y) = \begin{pmatrix} \frac{-y}{x^2+y^2} \\ \frac{x}{x^2+y^2} \end{pmatrix} \qquad rot(F) = 0 \qquad X = \mathbb{R}^2 \setminus \{(0,0)\}$$

$\Downarrow$

F È IRROTAZIONALE ; X NON È SEMPLICEMENTE CONNESSO

RESTRINGIAMOCI A $\Omega = \{ \text{semipiano} \ , \ y > \frac{\pi^2}{36} \}$

Ω s.connesso $\subset \Gamma$ $\Rightarrow$ F È CONSERVATIVO IN Ω

$\Gamma_1 = \{ x = \log(\frac{7}{4}) \ , \ (\frac{\pi}{6})^2 \le y \le (\frac{\pi}{2})^2 \}$ $\gamma_1(t) = (\log(\frac{7}{4}), t)$ $t \in [\frac{\pi}{6}, \frac{\pi}{2}]$

$\gamma_1'(t) = (0, 1)$

$$F(\gamma(t)) = \begin{pmatrix} \frac{-t}{\log^2(\frac{7}{4}) + t^2} \\ \frac{\log(\frac{7}{4})}{\log^2(\frac{7}{4}) + t^2} \end{pmatrix} \qquad \int_{(\frac{\pi}{6})^2}^{(\frac{\pi}{2})^2} \frac{\log(\frac{7}{4})}{\log^2(\frac{7}{4}) + t^2}\, dt \overset{(-1)}{=} 0{,}189223$$

⊖

$\Gamma_2 = \{ 0 \le x \le \log(\frac{7}{4}) \ , \ y = \frac{\pi^2}{4} \}$ $\gamma(t) = (t, \frac{\pi^2}{4})$ $t \in [0, \log\frac{7}{4}]$

$$F(\gamma(t)) = \begin{pmatrix} \frac{-\pi^2}{4} \\ t^2 + (\frac{\pi^2}{4})^2 \end{pmatrix} \qquad \int_0^{\log\frac{7}{4}} \frac{-\frac{\pi^2}{4}}{(\frac{\pi^2}{4})^2 + t^2}\, dt \overset{(-1)}{=} -0{,}122303$$

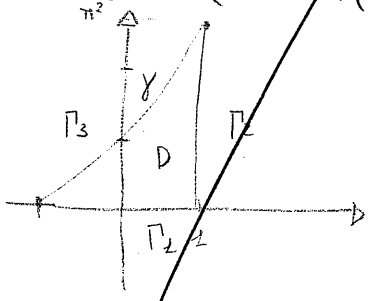
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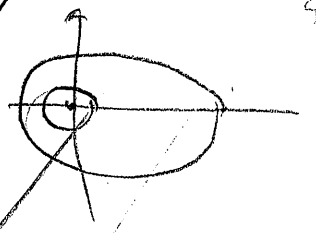
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$\gamma(t) = (\sin t \ , (t+\frac{\pi}{2})^2)$ $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\gamma(-\frac{\pi}{2}) = (-1, 0)$

$\gamma(\frac{\pi}{2}) = (1, \pi^2)$





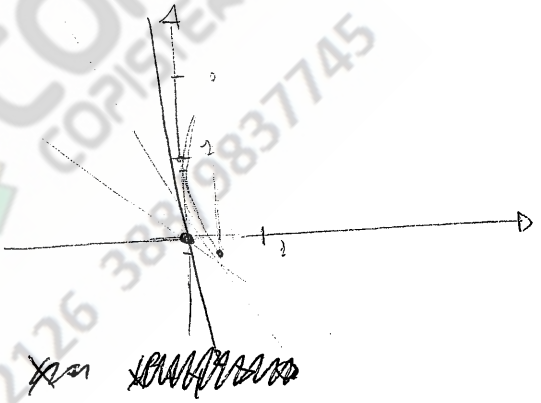
AREA $D = \iint_D 1 \, dx dy = L(F, \gamma)$

$\Gamma_1 = \{ -1 \le x \le 1 \ , \ y = 0 \}$ $\gamma(t) = (t, 0)$ $t \in [-1, 1]$

$L(F, \gamma_1) =$

$\gamma(t) = (\log(1+t^2) \ , t + \cos t)$ $t \in [-1, 1]$

$$F(x,y) = \left(\frac{x}{\sqrt{x^2+y^2}(1+\sqrt{x^2+y^2})} \ , \frac{y}{\sqrt{x^2+y^2}(1+\sqrt{x^2+y^2})} \right)$$



$\gamma(-1) = (\log 2 \ , -1 + \cos(-1))$

$\gamma(1) = (\log 2 \ , 1 + \cos 1)$

$\gamma(0) = (0, 1)$

$x^2+y^2 > 0$ $\Omega = \mathbb{R}^2 \setminus \{(0,0)\}$ NON SEMPL. CONNESSO

$rot(F) = 0$ $\Omega = \{ y \ge -100x \}$ F È CONSERVATIVO SU Ω XK Ω È SEMPLICEMENTE CONNESSO

$\Gamma = \{ x = \log 2 \ , \ -1 + \cos(-1) \le y \le 1 + \cos(-1) \}$

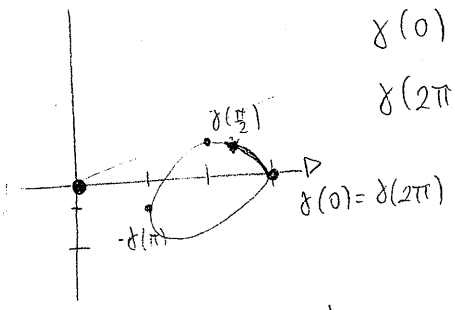
$\gamma(t) = (\log 2, t)$ $t \in [-1 + \cos(-1), 1 + \cos(-1)]$

$\gamma'(t) = (0, 1)$ $F(\gamma(t)) = \left(\frac{t}{\sqrt{\log^2 2 + t^2}(1+\sqrt{\log^2 2 + t^2})} \right)$

$\gamma(t) = (2+\cos t \,,\ \sin(t^2 - 2\pi t + 1)) \quad t \in [0, 2\pi]$

$\gamma(0) = (3, 0) \qquad \gamma(+\pi) = (+1 \,,\ \sin(-\pi^2))$ (5)

$\gamma(2\pi) = (3, 0) \qquad \gamma(\frac{\pi}{2}) = (2 \,,$

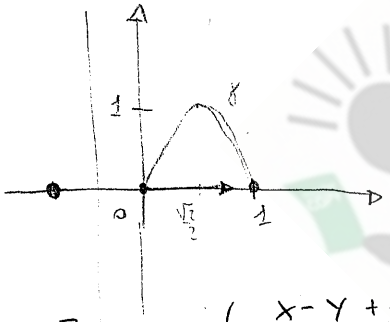


$F(x,y) = \begin{pmatrix} \frac{-y}{x^2+y^2} \\ \frac{x}{x^2+y^2} \end{pmatrix}$ $\quad rot(F) = 0$ IRROTAZIONALE

$X = \mathbb{R}^2 \setminus \{(0,0)\}$

$L(F,\gamma) = \iint_\Omega rot(F)\, dx\,dy = \iint_\Omega 0 \, dx\,dy = 0$

$\gamma(t) = (\sin t, \sin(2t)) \quad t \in [0, \frac{\pi}{2}]$



$\gamma(0) = (0,0)$ INIZ. $\qquad \gamma(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, 1)$

$\gamma(\frac{\pi}{2}) = (1, 0)$ FIN

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$F(x,y) = \begin{pmatrix} \frac{x-y+1}{(x+1)^2+y^2} \\ \frac{x+y+1}{(x+1)^2+y^2} \end{pmatrix}$ $\qquad X = \mathbb{R}^2 \setminus \{(-1,0)\}$

$rot(F) = 0$

X NON SEMPLICEMENTE CONNESSO

RESTRINGIAMOCI A $\Omega = \{x > -\frac{1}{2}\}$ PER CUI F È CONSERVATIVO IN Ω

$L(F,\gamma) = -L(F,\gamma_1)$

$\gamma_1 = \{0 \le x \le 1,\ y = 0\} \qquad \gamma(t) = (t, 0) \quad t \in [0,1]$

$\frac{t+1}{(t+1)^2} = \frac{1}{t+1} \Rightarrow \int_0^1 \frac{1}{1+t} = \log(1+t)\Big|_0^1 = \log 2$